STRATEGIC DISCLOSURE AND INVESTOR LOSS AVERSION [‡]

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Abstract

We examine the impact of loss aversion on firms' voluntary disclosure behavior. Incorporating investors with Prospect Theory preferences (Kahneman and Tversky 1979) into the classic voluntary disclosure setting of Dye (1985), we generate a novel, three-threshold equilibrium where managers voluntarily disclose 'mildly bad' but withhold 'mildly good' news. This asymmetric disclosure behavior is driven by managers rationally avoiding investor loss aversion associated with missing earnings expectations. This behavior is consistent with existing empirical evidence documenting that managers strategically guide down market expectations to avoid negative earnings surprises. The model predicts that the effect will be stronger in settings where the degree of loss aversion among investors is more pronounced and when the manager's information is more precise. We also find that outside these mild news regions, traditional disclosure predictions still hold. Namely, managers disclose 'very good' and withhold 'very bad' news because the influence of loss aversion diminishes farther away from the meet-or-beat threshold.

Keywords: Prospect Theory, loss aversion, voluntary disclosure, guidance

JEL Classifications: G40, G41, M40, M41

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1. Introduction

This paper examines how managers guide market expectations through strategic disclosure vis-à-vis behavioral investors. In the past half century, research has increasingly documented that market participants behave in ways that systematically and predictably deviate from the canonical von Neumman-Morgenstern Expected Utility Theory (EUT). A prevailing criticism of EUT is that it provides a *normative* description of economic agents rather than a *realistic* one. Of the plethora of non-EU theories that have subsequently emerged (Starmer 2000), Prospect Theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992) gained the most traction in all of theoretical foundation, experimental evidence, and empirical application. The past decades have seen Prospect Theory extend its reach to asset pricing (Barberis et al. 2001), contract theory (Dittmann et al. 2010), and industrial organization (Heidhues and Kőszegi 2014). This study forays into a novel area connecting this behavioral research to extant disclosure theory.

We develop a model that analyzes voluntary disclosure decisions of a potentially informed manager of a publicly traded firm who faces Prospect-Theoretic investors. Informally, Prospect-Theoretic investors are characterized by three attributes: (1) they derive utility from gains and losses (instead of absolute wealth) relative to a reference point (e.g., earnings expectations). That is to say, what matters to investor utility is not just the earnings outcome by itself but also how the earnings fare against the investors' prior expectations. (2) Their utility function is steeper in losses than in gains. In the context of earnings, loss-averse investors derive excess negative utility from missing earnings expectations compared to beating earnings expectations. For example, the absolute value of the utility lost over missing earnings expectations is greater than that gained from beating earnings expectations by the same amount. (3) Their utility function is concave in gains (or beating expectations) and convex in losses (or missing expectations). This "diminishing sensitivity" characteristic manifests as investors being acutely sensitive to small losses, just below the reference point of expectations. For instance, the marginal utility loss of missing earnings expectations by a cent is disproportionately worse than missing by an additional cent. When earnings are sufficiently far below expectations, investors' marginal utility loss asymptotes to the EUT loss. Marginal loss aversion asymptotes to zero as the losses mount towards infinity. These dynamics are mirrored in piecemeal empirical evidence on price reactions and other events regarding earnings forecasts and announcements. We later discuss this body of empirical literature and use it to motivate our modeling exercise: the goal is to take this refinement of investor utility seriously and learn about the manager's optimal disclosure rule in response.

As such, we analyze a model of rational managerial disclosure choice in the face of a rational but loss-averse stock market. The starting point of this exercise is the classic voluntary disclosure model with uncertainty about information endowment (Dye 1985, Jung and Kwon 1988). We modify the investor's utility, hence his pricing function, and consider the manager's optimal disclosure rule in equilibrium. The essential synopsis of the model is the following: this model has two periods. In the first period, the representative manager receives a noisy signal, with some probability, of the cashflow that materializes at the end of the second period. To investors, there is uncertainty as to whether the manager is informed or not. The manager chooses whether to disclose truthfully or to withhold the signal. The representative investor observes either the signal or the lack of disclosure, and Bayesian updates his expectation of the cashflow. He sets the interim price for the firm based on this expectation. In the second period, cashflows are realized and revealed to all. The investor evaluates this outcome against his latest prior - his updated expectation after observing the manager's disclosed signal or the lack of disclosure in the first period. Once again, he prices the firm - this time, based on certain information (i.e., period-end earnings). The manager has Ross-like utility (Ross 1977) and cares about interim firm value. He seeks to maximize the firm prices at both points in time. Caring about interim firm value is crucial for motivating the incentive to withhold bad news.[1] Interestingly, our Prospect-Theoretic investor

[1]Consistent with caring about interim firm value, Johnson and So (2018) document that managers with

prices the firm based on his asymmetric gain/loss utility. Later in this paper, we formalize the functional form and economic intuition. Informally, the investor cares about absolute *and* relative cashflows. As the cashflow meets or beats his expectation, he has positive utility over this "gain." And when the realized cashflow falls below, he has negative utility over the "loss," and he feels the losses more acutely than the gains. Or, in the words of Kahneman and Tversky (1979), "Losses loom larger than corresponding gains" for our investor.

Our analyses show that, in equilibrium, there are three thresholds (i.e., four signal regions) of the voluntary disclosure decision. Conditional on becoming privately informed, the manager discloses very positive signals and does not disclose very negative signals. This is consistent with extant theory. The novel finding is that there are two interior signal regions: *mildly negative* signals are strategically voluntarily disclosed to avoid disappointing investors on the announcement of cashflows. Similarly, *mildly positive* signals are strategically withheld to the same end.

The setting of the model is most analogous to voluntary managerial forecast/guidance ahead of earnings announcements. Numerous accounting and finance studies robustly document that median earnings surprises have shifted from a small negative (narrowly miss) to a small positive (narrowly beat) since the 1980s (Brown 2001). Matsumoto (2002) finds that a disproportionate percentage of firms beat analyst expectations by just a few cents. Additionally, Bartov et al. (2002) documents a substantial and persistent premium to meeting or beating analyst expectations. We empirically confirm that these phenomena - the asymmetry in small beats/misses near expectation and the asymmetry in market response to beating/missing by the same amount - persist through the current date (Figure 1 and Figure 2). The literature posits that managers are not merely passive spectators of the meet/beat game but instead active participants through two complementary channels. The first is earnings management: holding expectations constant, managers actively manipulate earnings such that some firms that would otherwise fall below expectations manages to rise

positive earnings announcement news move up their earnings announcements.

above. For example, Bhojraj et al. (2009) find that managers myopically engage in discretionary adjustments to beat analyst forecasts with low-quality earnings. The second is expectations management: holding earnings genuine, managers actively guide market expectations downward towards beatable targets. For example, Soffer et al. (2000) documents that managers strategically guide market expectations to avoid negative earnings surprises. Our study informs the latter channel. While earnings management has been explicitly modeled to explain how it is a sustained equilibrium outcome (Stein 1989, Guttman et al. 2006), less careful consideration has been afforded to expectations management or the walk-down to beatable targets.

Hence, the primary contribution of this study is to provide a dynamic narrative of earnings expectations management. Much of the extant work on disclosure considers a one-period game in which a manager discloses to a representative investor who prices the firm, and the game ends. In contrast, this paper speaks better to the disclosure practice by studying how a manager voluntarily discloses earnings forecasts to influence investor expectations and market reaction to subsequent earnings announcements. Thus, this paper is related to the theoretical literature on the disciplining role of financial reports on preceding voluntary disclosure (e.g., Stocken 2000a,Gigler and Hemmer 2001, Bertomeu et al. 2022). While their focus is on the properties of voluntary disclosure, our focus is on the behavioral reaction of investors to the earnings announcement and how managers use the preceding voluntary disclosure to shape that reaction.

Our paper also contributes to the literature on voluntary disclosure that examines why managers may voluntarily disclose bad news. Early disclosure models centered on the unraveling result (Grossman and Hart (1980), Grossman (1981), and Milgrom (1981)), which identifies a set of conditions under which an informed party will always voluntarily disclose private information. [2] However, empirically, we observe that managers disclose bad

[2]These conditions have led to many canonical disclosure models that closely examine violations of each premise. Beyer et al. (2010) provide an in-depth review of the theory literature. Broadly, non-disclosure equilibria exist when [1] disclosure is costly (Jovanovic 1982; Verrecchia 1983; Verrecchia 1990; Dye 1986;

news voluntarily, and prior literature has offered other explanations. For example, litigation risk may motivate firms to reveal bad news diligently (Skinner 1994; Kasznik and Lev 1995; Baginski et al. 2002; Cohen et al. 2020). Managers may also withhold good news before option grant dates to lower the exercise price of options (Aboody and Kasznik 2000, Richardson et al. 2004). These explanations often rely on stylized settings where managers have situation-specific incentives.

However, disclosure has yet to be analyzed in the context of investors with behavioral preferences, and this study bridges the gap between the disclosure literature and behavioral economics. Studies led by Tversky and Kahneman (1992) and Benartzi and Thaler (1995), and more recently Barberis et al. (2001) and Barberis and Thaler (2003), document how loss aversion is an important feature of investor utility. We cast a spotlight on this feature by developing a theory that can explain the empirical reality of managers' disclosure choices. By incorporating such Prospect-Theoretic investor preferences in a voluntary disclosure model, we gain important insights into the key economic trade-off of a manager when choosing his optimal disclosure rule: interim price and future disappointment. Our main finding that the choice to disclose is non-linear with respect to the news provides a rational foundation for the sizeable literature on expectations management.

The rest of the paper is organized as follows. Section 2 of this paper provides motivating empirics to the model. In section 3, we formally derive and analyze the baseline model. Section 4 discusses comparative statics. Section 5 considers model extensions. Section 6 provides thoughts on future work and concludes. All mathematical analyses and proofs are delegated to the appendices.

Lanen and Verrecchia 1987; Darrough and Stoughton 1990; Feltham and Xie 1992), when [2] there is uncertainty about the endowment of private information (Dye 1985; Jung and Kwon 1988; Penno 1997; Pae 2002), [3] disclosure is unverifiable or includes misreporting (Crawford and Sobel 1982; Gigler 1994; Stocken 2000a; Korn 2004, Beyer and Guttman 2012; Einhorn and Ziv 2012), [4] the investors' response to disclosure is dubious (Dutta and Trueman 2002; Suijs 2007), and finally, when [5] the firms can ex-ante, credibly commit to a disclosure policy.

2. Motivating Empirical Facts

Our theory exercise relates to a large body of empirical literature on earnings announcements and managerial guidance.

A prominent finding in the accounting literature is that there is an exceptionally high occurrence of small earnings increases and a correspondingly high occurrence of small earnings decreases (Burgstahler and Dichev 1997). This observation is frequently made in accompaniment of the finding that there exists a discontinuity in stock returns for firms that beat vs. miss by a small margin. For instance, Bhojraj et al. (2009) rigorously document that firms myopically endeavor to beat (or avoid missing) benchmarks by a cent. We replicate this finding for the post-SOX period and find that this discontinuous asymmetry in returns around small gain and small losses persists through the current day.

[Insert Figure 1]

Earnings announcements that beat market expectations by 1 cent have a higher average return of 1.6 percent compared to those that miss by 1 cent (Figure 1 Panel A). We find similar patterns for earnings announcement returns when surprise earnings is scaled by lagged firm share price (Figure 1 Panel B). This discontinuity in price response around the reference point of market expectation echoes the sentiment expressed by CEOs in Graham et al. (2005)'s survey, that managers perceive the capital market penalty to missing short term earnings benchmarks to be severe.

Our paper squares these two observations - the positive asymmetry in density of earnings surprises, and the negative asymmetry in market response to earnings surprises - through the lens of expectations management. In our model, the disproportionate penalty around small misses arise from investor loss aversion. Managers internalize and rationally cater to this investor preference in order to maximize stock price. In doing so, the manager strategically discloses mildly negative and withholds mildly positive news. Among other finidngs, in equilibrium, managers experience more small beats.

A large literature discusses the discontinuity through the lens of earnings management. Guttman et al. (2006) examine a model in which a manager trades-off benefits from boosting interim stock price and costs of manipulating earnings. They find that multiple equilibria can exist. In one equilibrium, discontinuities/kinks in reported earnings emerge endogenously as a result of a partial pooling strategy. Earnings management is a well-accepted explanation for the asymmetric beat/miss phenomenon. However, meeting expectations through earnings management, regardless of whether it is through accruals or real activities, can be costly. Prior studies have documented that earnings management is associated with sub-optimal investment decisions and/or future performance declines (McNichols and Stubben 2008; Bhojraj et al. 2009). In addition, some firms may face constraints in their capacity for earnings management (Barton and Simko 2002). When it comes to managers optimizing towards the goal of beating benchmarks, we view earnings management as a complementary channel to strategic disclosure. But given aforementioned costs and constraints of earnings management, one may wonder to what extent it can be fully responsible for thephenomenon where markets appear to consistently under-expect earnings. And, consequently, can alternative mechanisms be a factor?

In this paper, we formalize the argument that expectations management can play a role in the systematic asymmetry between beating and missing. We posit that managers rationally cater to investors with asymmetric gain-loss utility. In doing so, managers will disclose mildly negative and withhold mildly positive signals. This strategic disclosure avoids investor loss aversion associated with missing earnings. Bhojraj et al. (2009) rigorously documents a discontinuity in stock returns for firms that beat vs. miss by a cent. Our motivating empirics below document that managers respond to this feature of earnings announcements.

The sample spans all firms post-SOX (Dec 2004 to Dec 2022) with earnings guidance.

We further restrict to firm-quarter observations where the firm has at least a market cap of 50 million and a price of $5. In cases where there is more than one guidance for a given earnings period, we choose the guidance nearest to the quarterly earnings announcement. To compute guidance surprise, we require that the firm have analyst coverage with consensus EPS data. These data are standard from I/B/E/S and CRSP-Compustat. The resulting sample has 2,893 firms with an average participation of 11.7 years and approximately 34,000 firm-earnings observations. We measure guidance surprise with respect to analyst consensus before guidance:

$$GuidanceSurprise_{i,t} = \frac{Guidance_{i,t} - Consensus_{i,t}}{Price_{i,t-1}} \tag{1}$$

where i indexes firms and t indexes quarter t. Similarly, we define earning surprise as:

$$EarningsSurprise_{i,t} = \frac{Earnings_{i,t} - Consensus_{i,t}}{Price_{i,t-1}} \tag{2}$$

Figure 2 illustrates the propensity of managers to engage in strategic guidance. The distribution of earnings surprise (blue histogram) is overlaid on the distribution of guidance surprise (red histogram). To focus on the strategic guidance about mildly positive and mildly negative signals, we truncate the data at the top and bottom 0.5%. Guidance within this narrow region is negative 73% of the time and positive 27% of the time (red histogram). Managers are more than twice as likely to provide mildly negative guidance than mildly positive guidance.

[Insert Figure 2]

Conditional on providing guidance, managers are more likely to mildly beat than miss analyst expectations. Within this narrow region, managers beat earnings 72% of the time and miss earnings 28% of the time (blue histogram). This is consistent with the literature finding that managers are more likely to mildly beat than mildly miss earnings expectations.

While most of prior studies have focused on how managers achieve this through manipulating earnings, in this paper we take the approach of examining the managers' optimal voluntary disclosure strategy. The propensity of managers to strategically guide earnings is empirically well-documented but is inconsistent with existing theories of disclosure. Thus, an important contribution of this paper is to bridge this gap and rationalize this strategic disclosure. We do so by modifying classic disclosure theory to incorporate Prospect-Theoretic preferences. These preferences result in prices responding more to negative earnings announcements than negative earnings forecasts. Our theoretical model yields a novel interior region where managers disclose mildly bad news and withhold mildly good news in equilibrium. These rational, strategic disclosure regions are consistent with the empirical reality of how managers walk down market expectations to achieve earnings targets.

3. Baseline Model

Environment and Timeline

In our baseline model, a rational manager facing loss-averse investors makes a voluntary disclosure choice. The model expands on the setting of Dye (1985) to include Prospect Theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992) preferences within the representative investor's utility function. The model features a manager, m, and a representative investor, i. Figure 3 shows the sequence of events.

[Insert Figure 3]

At $t = 0$, the manager receives a noisy signal of earnings, s, with probability q. We assume that the true earning, x, follows a normal distribution. Its mean is normalized to 0 and its variance is σ_X^2, and we denote its density function $f(x)$. This distribution is common prior to both the manager and the investor. The manager's signal is noisy about the true earning e: $s = x + e$, where $e \sim N(0, \sigma_e^2)$ is a white-noise term and is independent of x. The manager cannot credibly convey whether he has received the signal. Further, he is no more informed beyond the common prior. The manager chooses to either truthfully disclose his private signal (s) or withhold the signal ($\emptyset$).

The manager has Ross-like utility (Ross 1977) and values interim and final stock prices. As such, he chooses the action that ex-ante maximizes the sum of stock prices in both periods. His utility function is:

$$U^m = P_0 + P_1 \tag{3}$$

where P_t denotes the stock price. For simplicity, we suppress the discount factor.

The Prospect-Theoretic investor derives (EUT) utility from earnings but further derives negative utility when earnings fall below his expectation:

$$U^i(x, E[\tilde{x}|I]) = x - \gamma \cdot 1_{x<E[\tilde{x}|I]} \tag{4}$$

where γ is the loss-aversion coefficient, and the information set $I \in \{s, \emptyset\}$ denotes the manager's action from the investor's perspective. In the next subsection, we discuss this utility function in more technical detail.

At $t = 0$, the investor prices the firm's expected future earnings and expected future loss aversion, conditional on managerial disclosure (s) or non-disclosure ($\emptyset$). At $t = 1$, the investor prices the firm's realized earnings with a disappointment penalty if it is below expectation. Note that although the investor is behavioral in his loss aversion, he is time-consistent in his pricing of future expected loss aversion.

$$P_t = \begin{cases} E[U^i(\tilde{x})|I] & \text{if } t = t_0 \\ U^i(\tilde{x}) = x - \gamma \cdot 1_{E[x<\tilde{x}|I]} & \text{if } t = t_1 \end{cases} \tag{5}$$

When choosing whether to disclose his signal, the manager trades off the value of interim prices with the probability of disappointing investors. For positive signals, the manager may disclose and enjoy a high interim price at the cost of a greater probability of disappointment. For negative signals, the manager may disclose and suffer a low interim price for the gain of a decreased probability of disappointment.

Investor Loss Aversion

The investor's utility function is based on Tversky and Kahneman (1992)'s Prospect Theory. The authors document in their papers that, when evaluating risky gambles, economic agents systematically make decisions that exhibit loss aversion, reference dependence, and diminishing sensitivity, among other psychological heuristics. Formally, in their paper, investor utility takes the functional form of:

$$U(x) = \begin{cases} x^{\alpha} & \text{for } x \geq 0 \\ -\gamma(-x)^{\beta} & \text{for } x < 0 \end{cases} \tag{6}$$

where x denotes the outcome (gain). The loss aversion coefficient, γ, captures the intuition that investors are distinctly more sensitive to losses than gains. The disappointment penalty scales proportionally with γ. β disciplines the curvature to the loss aversion. Investors have slower diminishing sensitivity to loss aversion when the curvature is flatter (a larger β).

Following the publication of Prospect Theory, financial economists have applied these preferences in numerous settings, especially loss aversion. For example, Benartzi and Thaler (1995) suggest loss aversion as a main driver of the equity premium. Barberis and Thaler (2003) provide an extensive survey of capital market studies with loss-averse investors. Loss aversion is a feature of the representative investor in our managerial disclosure setting. We specify investor utility as in equation (4):

$$U^{i}(x, E[\tilde{x}|I]) = \begin{cases} x & \text{for } x \geq E[\tilde{x}|I] \\ x - \gamma & \text{for } x < E[\tilde{x}|I] \end{cases} \tag{7}$$

In the baseline model, we consider a discontinuous loss to the investor's utility for earnings missing the expectation. Bhojraj et al. (2009) provide evidence of a discontinuity in stock price returns about earnings just above and below expectations. This earnings discontinuity is the empirical micro-foundation of the investor's utility function. The investor's utility has two components: the first part is linear in the firm's cash flows, and the second part, $-\gamma \cdot 1_{E[x<\tilde{x}|I]}$, is the loss penalty associated with earnings missing expectation. Earnings below expectations is penalized by γ. Note that loss aversion is with respect to rationally Bayesian updated investor cash flow expectations. This is context-appropriate given the earnings setting. Tversky and Kahneman (1992) set the reference point to 0 due to their experimental setting of risky gambles. In their paper, a gain is defined as cash winnings above \$0 and a loss is the opposite. We choose expectations as a reference point, consistent with the approach in Koszegi and Rabin (2006). In model extensions, we analyze the effect of including curvature to loss aversion (β) as in Tversky and Kahneman (1992)'s original utility function.

Section 4 extends the model to consider a continuous version of Prospect-Theoretic investor utility. This extension is not analytically tractable to the extent of the baseline model. However, we numerically simulate the equilibrium and its characteristics.

Equilibrium

In equilibrium, the manager optimally discloses subject to the trade-off between interim prices and the probability of disappointing loss averse investors. Given this disclosure rule, investors form expectations about the signal received by managers and price the firm. Formally, we define the equilibrium below.

An equilibrium consists of the manager's disclosure strategy $R(\cdot)$ and the price function $P(\cdot)$ such that:

(i) Given the price function $P(\cdot)$, the informed manager's disclosure strategy $R(\cdot)$ maximizes his utility:

$$I(s) \in \arg \max_{I \in \{s, \emptyset\}} P_0 \left(I \left(s\right)\right) + E \left(P_1 \left(I \left(s\right), x\right)\right)$$

(ii) Given the disclosure rule $I(\cdot)$, prices $P(\cdot)$ satisfy:

$$P_1(I, x) = U^i \left(x, E\left(\tilde{x}|I\right)\right) = x - \gamma \cdot 1_{x < E[\tilde{x}|I]}$$

$$P_0(I) = E\left[U^i \left(x, E\left(\tilde{x}|I\right)\right)\right] = E\left(\tilde{x}|I\right) - \gamma \cdot Pr\left(\tilde{x} < E\left(\tilde{x}|I\right)|I\right)$$

where $I\left(s\right) \in \{s, \emptyset\}$.

And we assume that when the manager is indifferent between disclosure or withholding, he withholds.

4. Analysis

To solve for the equilibrium, we first analyze the manager's disclosure choices given their conjecture about the investor's beliefs regarding non-disclosure firms. We can assume that the investor conjectures the non-disclosing firms have an average firm value $E_i(x|I = \emptyset) = \kappa$ and will later disappoint with probability $Pr_i(x < \kappa) = \delta$. Then we derive the properties of firms that withhold disclosure and finally solve the equilibrium, i.e., (κ, δ), by noting that, in equilibrium, the manager's conjectures must be consistent with the investor's belief and stock pricing.

4.1. Stock Price

At $t = 1$, cash flow x is revealed, and the investor prices the firm at its cash value minus a disappointment penalty if the cash flow falls short of the investor's expectation.

$$P_1(x, I) = U_i\left(x, E\left(\tilde{x}|I\right)\right) = x - \gamma \cdot 1_{x<\kappa}$$

Going back to $t = 0$, the investor prices the firm at the expected value minus the expected disappointment, i.e.,

$$
\begin{aligned}
P_0(I) &= E_i\left[P_1(x, I)\right] \\
&= E_i(x|I) - Pr_i\left(x < E\left(\tilde{x}|I\right)\right) \\
&= \kappa - \gamma\delta
\end{aligned}
$$

Both $P_1(x, I)$ and $P_0(I)$ depend on the investor's information set. From the investor's perspective, $P_0(I)$ equals $E_i\left[P_1(x, I)\right]$, i.e., the $t = 1$ payoff including the loss-aversion penalty. However, the manager may have different information set from the investor if he chooses to withhold. Next, we turn to the manager's disclosure choice.

4.2. Manager's Disclosure Choice

If the informed manager discloses $I = s$, then the investor's posterior belief is updated to $E_i(x) = \frac{2\sigma_X^2}{\sigma_X^2+\sigma_e^2}s$, and she will be disappointed with probability 1/2. Thus the manager's expected payoff for disclosure is:

$$
\begin{aligned}
P(I = s) &= E\left(P_1(I = s, x)\right) + P_0(I = s) \\
&= 2P_0(I = s) = 2E(x|s) - \gamma \\
&= \frac{2\sigma_X^2}{\sigma_X^2 + \sigma_e^2}s - \gamma
\end{aligned}
$$

On the other hand, if the informed manager withholds, his payoff would depend on the investor's conjecture about non-disclosure:

$$
\begin{aligned}
P(I = \emptyset) &= P_0(I = \emptyset) + E_m(P_1(I = \emptyset, x)) \\
&= \kappa - \gamma\delta + E_m(x) - \gamma Pr_m(x < \kappa)
\end{aligned}
$$

In equilibrium, the informed manager discloses if and only if the payoff from disclosure is higher than (or equal to) non-disclosure, i.e., $g\left(s,\kappa,\delta\right)\geq 0$:

$$\begin{aligned} g\left(s,\kappa,\delta\right) &= P\left(I=s\right)-P\left(I=\emptyset\right) \\ &= (2E(x|s)-\gamma)-(\kappa-\gamma\delta+E_m(x)-\gamma Pr_m(x<\kappa)) \\ &= E\left(x|s\right)-E\left(x|\emptyset\right)+\gamma\left(\Pr\left(x<\kappa|s\right)+\delta-1\right)>0 \end{aligned}$$

The above equation shows that the incentive to disclose voluntarily may not be monotonically increasing with s, i.e., the signal that informed managers observe. The sign of $\frac{\partial g(s,\kappa,\delta)}{\partial s}$ is ambiguous due to the two opposing effects. First, we can see that $\frac{\partial E(x|s)}{\partial s}>0$, that a higher signal indicates a higher firm value and thus can lead to a higher price under disclosure. Second, $\frac{\partial \Pr(x<\kappa|s)}{\partial s}$ decreases with s, suggesting that a higher signal indicates a smaller possibility of disappointment under no disclosure, making withholding more attractive.

We start by showing that this voluntary disclosure game has at least an equilibrium.

Proposition 1. *Suppose the investor believes the withholding firms have an expected value of κ and a probability of disappointment δ. The manager's optimal response is to disclose if $g(s,\kappa,\delta)>0$. Under the optimal response strategy, denote $h\left(\kappa,\delta\right)$ to be the expected value and probability of disappointment for firms withholding disclosure, i.e., $h\left(\kappa,\delta\right)=\left(\kappa',\delta'|\kappa,\delta\right)$, where $\kappa'=E(x|I=\emptyset,\kappa,\delta)$ and $\delta'=E(Pr(x<\kappa)|I=\emptyset,\kappa,\delta)$. There exists at least one fixed point $\left(\kappa^{*},\delta^{*}\right)$ such that $h\left(\kappa^{*},\delta^{*}\right)=\left(\kappa^{*},\delta^{*}\right)$.*

Proof. All proofs can be found in the appendix. □

Proposition 1 shows that at least one equilibrium exists. The mapping h gives the expected firm value and probability of disappointment when the investor's belief regarding non-disclosure firms is κ,δ. Our proof applies the Brouwer fixed-point theorem and goes into two steps. We first show that the probability of disappointment $\delta\in[0,1]$, and the expected value of withholding firms, $|\kappa|$, are bounded. Next, we show that the mapping is continuous, and thus the Brouwer fixed-point theorem can be applied to ensure at least one equilibrium.

Once we establish the existence of the equilibrium, we turn to characterize the equilibrium. The informed manager's disclosure payoff may not increase monotonically in his private signal, leading to a novel, three-threshold equilibrium.

Lemma 1. *For any investor's belief (κ, δ), there exists a minimum of one solution to $g(s, \kappa, \delta) = 0$ and a maximum of three solutions.*

This lemma shows that the informed manager's disclosure strategy would be a conventional one-threshold or three-threshold equilibrium, which we will characterize next.

Proposition 2. *In a three-threshold equilibrium,*

1. *Denote $g(s, \kappa, \delta)$ to be the difference in the informed manager's expected utility between disclosing and withholding. The manager's optimal reporting strategy is*

$$I(s) = \begin{cases} s, & \text{if } s_H \leq s. \\ \emptyset, & \text{if } s_M \leq s < s_H. \\ s, & \text{if } s_L \leq s < s_M. \\ \emptyset, & \text{if } s < s_L. \end{cases} \tag{8}$$

   where $s_L < s_M < s_H$ are the three roots to $g(s, \kappa, \delta) = 0$. Furthermore, At $s = s_M$, we have $g'(s) < 0$.

2. *The pricing function $P(\cdot)$ takes the form:*

   (a) $P_0(I = s) = E(x|I = s) - \frac{\gamma}{2}$ and $P_1(I = s) = x - \gamma 1_{x<E(x|I=s)}$.

   (b) $P_0(I = \emptyset) = \kappa - \gamma\delta$ and $P_1(I = \emptyset) = x - \gamma 1_{x<\kappa}$

3. *The investor's belief is consistent with the informed manager's disclosure strategy, i.e.,* $\kappa = E[x|I = \emptyset]$ *and* $\delta = \int_{-\infty}^{E[x|I=\emptyset]} f(x|I = \emptyset)dx$.

Figure 4 illustrates $g(s, \kappa, \delta)$ for a set of parameter values and provides economic intuition of Proposition 1.

$$g(s, \kappa, \delta) = P_0(I = s) - P_0(I = \emptyset) - \left(\frac{\gamma}{2} - \gamma \int_{-\infty}^{\kappa} f(x|I = s)dx\right) \tag{9}$$

As discussed earlier, the informed manager's incentive to disclose is not monotonically increasing in s. This is because he faces the trade-off between the interim price effect $(P_0(s) - P_0(\emptyset))$ and the expected disappointment effect $(\frac{\gamma}{2} - \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx)$. While the benefit of disclosure increases with a more favorable private signal s, so does the benefit of withholding: a more favorable private signal means investors are less likely to be disappointed, but only if it is withheld. Once disclosed, the investor would update her posterior and reset the reference point. Note that, unlike the investor who estimates disappointment in $f(x|I = \emptyset)$, the manager calculates expected disappointment in $f(x|I = s)$, since he privately knows the signal.

[Insert Figure 4]

For $E[x|s] < E[x|\emptyset]$, the manager is more likely to disappoint investors if he withholds disclosure $(\frac{\gamma}{2} < \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx)$. However, if he does disclose, then interim prices are **lower** $(P_0(\emptyset) > P_0(s))$. For very low signals, i.e., $s < s_L$, the interim price effect dominates, and the manager chooses to withhold. For mildly low signals $s_L \leq s < s_M$, the disappointment effect dominates, and the manager discloses mildly bad news. This disclosure region is the novel outcome of incorporating investor loss aversion into the canonical disclosure model. In this region, managers disclose a signal worse than what the market would infer from no disclosure. The manager discloses mildly bad news to avoid investors' loss aversion at earnings announcements.

On the other hand, when $E[x|s] > E[x|\emptyset]$, the manager is less likely to disappoint investors if he does not disclose ($\frac{\gamma}{2} > \gamma \int_{-\infty}^{E[X|I=\emptyset]} f(x|s)dx$). However, if he does disclose, then interim prices are **higher** ($P_0(\emptyset) < P_0(s)$). For very high signals $s > s_H$, the interim price effect dominates, and the manager does disclose. For mildly high signals $s_M \leq s < s_H$, the disappointment effect dominates, and the manager withholds disclosure. Interestingly, in this region, the signal withheld by the managers is better than what the market would infer from no disclosure. The manager withholds such mildly good news to avoid earnings announcement loss aversion strategically.

To lend tactile scenarios to the four regions in Figure 4, consider the following scenarios. In one extreme case, if a manager learns that the firm will be close to bankruptcy by the end of the quarter, i.e., $s < s_L$, this model predicts that the manager will not provide guidance about the going concern risk; instead, he will delay disclosure until he has to do so. The value of a high interim stock price outweighs the disappointment to come. Conversely, if a manager learns that the firm will have outstanding quarter-end earnings, i.e., $s > s_H$, the model predicts that the manager will disclose this information, increasing interim stock price. This disclosure increases investor expectations but raises the risk of disappointment. However, the high interim stock price outweighs the risk of disappointment. Overall, the model predicts that the manager withholds very bad news but discloses very good news, consistent with predictions generated by most extant disclosure models. Absent loss aversion, a partial disclosure equilibrium where managers disclose good news and do not disclose bad news is well-established in extant literature (Dye 1985; Jung and Kwon 1988; Penno 1997; Pae 2002).

The novel prediction of our model is about the two regions in the middle. Consider the case where the manager has mildly positive news, such as the firm will slightly beat analyst expectations, i.e., $s_M \leq s < s_H$. The model predicts that the manager will not disclose the good news. Intuitively, a small price increase today is not worth the probability of later missing earnings expectations and incurring a disappointment penalty. Conversely, when a manager has mildly bad news, i.e., $s_L \leq s < s_M$, the manager in our model will strategically choose to disclose the bad news. Doing so will incur an interim period loss but reduces the likelihood of disappointing investors at the earnings announcement.

Proposition 3. *A necessary but insufficient condition for the existence of three-threshold equilibrium is* $\frac{\gamma}{(\sigma_{x,s})\sqrt{2\pi}} \geq 1$, *where* $\sigma_{x,s} = \sqrt{\frac{1}{\frac{1}{\sigma_x^2}+\frac{1}{\sigma_s^2}}}$.

If $\frac{\gamma}{(\sigma_{x,s})\sqrt{2\pi}} \leq 1$, *then the equilibrium is a single disclosure threshold equilibrium:*

$$I(s) = \begin{cases} S, & \text{if } s \geq s'. \\ \emptyset, & \text{if } s < s'. \end{cases} \tag{10}$$

where s' *is the single root to* $g(s, \kappa, \delta)$. *The pricing function* $P(\cdot)$ *takes the form:*

1. $P_0(I = s) = E(x|I = s) - \frac{\gamma}{2}$ *and* $P_1(I = s) = x - \gamma 1_{x<E(x|I=s)}$.

2. $P_0(I = \emptyset) = \kappa - \gamma\delta$ *and* $P_1(I = \emptyset) = x - \gamma 1_{x<\kappa}$

The investor's belief is consistent with the informed manager's disclosure strategy, i.e., $\kappa = E[x|I = \emptyset]$ *and* $\delta = \int_{-\infty}^{E[x|I=\emptyset]} f(x|I = \emptyset)dx$.

This proposition identifies a necessary condition for the existence of the three-threshold equilibrium. We can examine g'(s) to see the intuition, that is, how the disclosure incentive varies with the private signal. If $g'(s)$ is always positive, then there will be at most only one root to the equilibrium condition $g(s, \kappa, \delta) = 0$, and the three-threshold equilibrium would not exist. In that case, only the one-threshold equilibrium exists in which managers only disclose sufficiently bad news.

Note that we have left the two-threshold equilibrium unexplored. The two-threshold equilibrium is the special case of a three-threshold equilibrium in which two roots are identical.

Proposition 4. *The equilibrium is unique and has three thresholds when the following condition holds:*

1. *if γ is sufficiently large, and*
2. *if q is sufficiently small.*

This proposition identifies when the novel three-threshold equilibrium exists and its uniqueness. First, it exists when the disappointment penalty is sufficiently large and the manager's signal is sufficiently precise. We can prove by contradiction that the single-threshold equilibrium does not exist in that case. Suppose the equilibrium is a single-threshold one; then the manager with $g(s, \kappa, \delta) = 0$ would be indifferent between disclosure and withholding. However, for managers with signals close to the investor's average belief, while their payoffs from disclosure are similar, those right above it will incur a much smaller penalty from non-disclosure than those right below. As a result, those above (below) would be incentivized to disclose (withhold).

The intuition is that the informed manager with signals around investors' belief about the average non-disclosure firm faces a steep change in the expected disappointment penalty. The effect is more potent if the penalty is larger or the signal is more precise. Since higher signals significantly decrease the chance of disappointment with no disclosure, the manager slightly above (below) the threshold finds it more beneficial to disclose (withhold).

Secondly, the three-threshold equilibrium exists when q is sufficiently small, i.e., when $q \to 0$. The intuition is that when the manager is more likely to be uninformed, the non-disclosure firms are less likely to be strategic withholding, and the investor's belief about non-disclosure firms is higher. This makes the consideration for raising investors' expectations less critical and the incentive for avoiding disappointment more so. As a result, the firms with mild good (bad) news withhold (disclose).

5. Comparative Statics

Thus far, we have shown that a parsimonious voluntary disclosure model in which investors are loss averse can explain the pattern of managers guiding down market expectations before earnings announcements. This section presents comparative statistics concerning several main model parameters: the degree of investors' loss aversion (γ), the signal precision of the manager's private information (σ_e), and the probability of the manager receiving a signal (q). Varying these model features has directional and magnitudinal effects on the equilibrium.

Panels A through C in Figure 5 plot the optimal disclosure policy for variants to Figure 4 (Parameters: $q = 0.5, \sigma_X = 1, \sigma_e = 0.1, \gamma = 1$). In each figure, the y-axis plots the net managerial utility of disclosing (i.e., his utility from disclosing minus his utility from not disclosing). For values above 0, the manager has the incentive to disclose. The x-axis plots the private signal received by the manager.

Central to the incentive of strategic disclosure is loss aversion. To sustain a three-threshold equilibrium as described in Proposition 1, we know by Corollary 1, that $\frac{\gamma}{\sigma_{x,s}\sqrt{2\pi}} > 1$: γ needs to be sufficiently large. Investor loss aversion incentivizes managers to disclose mildly bad news and withhold mildly good news. As $\lim_{\gamma\to 0}$, the solution collapses to a single-threshold equilibrium (Corollary 2). The strategic disclosure region $s_L \leq s \leq s_H$ increases in the degree of loss aversion γ. Figure 5 Panel A illustrates the importance of a sufficiently large γ to incentivize strategic disclosure. The optimal disclosure policies for $\gamma = 0$ is the canonical Dye (1985) equilibrium of disclosing good news and withholding bad news. The manager who is privately informed would discloses his signal s only if $P_0(s) \geq P_0(\emptyset)$. For $\gamma = 1.5$, the manager discloses mildly bad news and withholds mildly good news.

To strategically avoid disappointing investors, managers need sufficiently precise private signals. To sustain a three-threshold equilibrium as described by Proposition 1, we know by

Corollary 3 , $\frac{\gamma}{\sigma_{x,s}\sqrt{2\pi}} > 1$: $\sigma_{x,s}$ needs to be sufficiently small. Uncertainty about earnings (X) conditional on a signal decreases in the precision of private signals (σ_e). As $\lim_{\sigma_e \to \infty}$, the solution collapses to a single-threshold equilibrium (Corollary 2). Figure 5 Panel B illustrates the importance of sufficiently precise private signals for strategic disclosure. Intuitively, without a sufficiently precise signal, the manager is less capable of implementing the optimal strategy whereby he maximizes stock price by minimizing investor loss aversion. The closer the manager knows the true probability of investor disappointment, via a precise signal, the more he is able to execute the optimal disclosure with knife-edge precision. With a very noisy signal ($\sigma_e = 0.5$), the manager revises his earnings expectation less ($\frac{\partial}{\partial \sigma_e} \frac{\sigma_X^2}{\sigma_X^2 + \sigma_\epsilon^2} < 0$). The equilibrium has a one-threshold reporting rule, but with additional curvature, the slope of $g(s)$ is flat nearby the threshold. The flat slope reflects the countervailing effect of disappointment risk, which is important for signals near the threshold, but not for signals far away. In contrast, with a very precise signal ($\sigma_e = 0.01$), the slope of managerial utility about the threshold becomes steeper. This feature captures the discontinuity of loss aversion. With a precise signal, the manager knows with high confidence whether or not he will disappoint investors. Therefore, his utility reflects the discontinuous gains from strategic guidance.

Finally, the probability of receiving a signal impacts the market's inference about a manager not disclosing any signal. This parameter impacts the asymmetry of the strategic disclosure regions. As the number of managers who do not receive a signal decreases, the pool of non-disclosing managers comprises more negative signals. This worsening of the pool incentivizes managers to disclose mildly negative news and deters managers from withholding mildly good news. For a more formal characterization of this comparative static, see Appendix 1.7. This asymmetric effect on the two interior disclosure regions is apparent in Figure 5 Panel C for a high probability of signal ($q = 0.9$) compared to a low probability of signal ($q = 0.1$).

[Insert Figure 5]

6. Model Extensions

The baseline model is intentionally parsimonious to capture strategic earnings guidance with minimal deviation from standard disclosure theory. The model elucidates the economic trade-off between higher interim stock prices and the risk of disappointing loss-averse investors. Within this section, we consider a variety of model extensions including, but not limited to, a non-closed form model using the general functional form of Prospect Theory utility, time varying loss aversion, and manegarial myopia.

True to the earnings announcement setting, we characterize a discontinuous loss-aversion penalty associated with missing earnings expectations. However, suppose we apply the general Prospect-Theoretic utility function. Tversky and Kahneman (1992)'s Cumulative Prospect Theory utility includes loss aversion and diminishing sensitivity. Utility diminishes steeply for earnings just below market expectations. For earnings far below expectations, the marginal loss-aversion asymptotes to zero. For earnings above and far below expectations, the slope of the utility function converges to linearity in cash flows. Formally, the investor utility function is

$$U^i(x) = \begin{cases} x & \text{for } x \geq E[X] \\ x - \gamma(E[X] - x)^\beta & \text{for } x < E[X] \end{cases} \tag{11}$$

where γ is the coefficient on the magnitude of loss aversion and β varies the curvature of loss aversion. Figure 6 illustrates how investor utility varies with $\gamma \in \{0, 1, 2\}$ and $\beta \in \{0.1, 0.3\}$ for $E[X] = 0$ and $x \in (-2, 2)$.

[Insert Figure 6]

Unlike the baseline model, the three-threshold reporting equilibrium of Proposition 1 is not analytically tractable in this setting. However, the manager faces the same economic forces when determining the optimal reporting strategy. Missing earnings is costly because of loss aversion. Investor utility is sharply decreasing for earnings below expectations. As with the baseline model, a larger loss aversion coefficient γ will increase the strategic disclosure region (see Figure 5 Panel A). Managers will withhold more positive signals and disclose more negative signals when loss aversion is severe. The novel parameter is β, which determines the curvature of the loss aversion. A flatter curvature to loss aversion corresponds to slower diminishing sensitivity (a larger β). Flatter curvature increases the set of negative signals that are disclosed, but decreases the set of positive signals that are withheld. The economic intuition is that loss associated with a near miss shrinks with greater curvature to loss aversion. Therefore, there is less strategic gain from withholding mildly positive news to avoid near-misses to earnings. Similarly, the marginal loss aversion from missing earnings converges to zero more slowly. Therefore, the manager strategically discloses a larger range of mildly bad signals to avoid loss aversion. In contrast, loss aversion is discontinuous as β approaches 0. The extension nests the baseline model. A steeper curvature increases the set of positive signals withheld and decreases that of negative signals disclosed. Figure 5 provides an example illustrating the effect of varying loss aversion curvature.

[Insert Figure 7]

In Figure 7, the baseline case example has a $\beta = 0.1$. All else equal, steeper curvature to loss aversion $\beta = 0.05$ decreases the negative signal disclosure region ($g(s) > 0$ region nearby -0.5) and increases the positive signal withholding region ($g(s) < 0$ nearby 0). In contrast, a flatter curvature to loss aversion $\beta = 0.2$ does the opposite.

An important feature of the model is that loss aversion occurs during the earnings announcement period. Suppose there was also loss aversion when managers release earnings guidance. Note that this is not an issue of time consistency of the earnings announcement loss aversion. Expected future loss aversion is rationally priced by investors. However, how would loss aversion toward managerial guidance (signal disclosure) impact the model of strategic guidance? If loss aversion is equally severe for guidance and earnings announcements, the manager cannot avoid loss aversion through guidance. This cuts against the core economic trade-off of the model. The underlying assumption of our model is that loss aversion is more severe for earnings announcements than guidance. This idea that losses hurt more in reality than in expectation is consistent with prior literature. Shefrin and Statman (1985) show that investors treat realized vs. unrealized gains/losses differently. Though imperfect, we analogize that negative earnings surprises at announcement presents a more realized loss than a negative managerial forecast revision. Moreover, prior accounting research has shown that negative earnings surprises being more costly than negative forecast revisions (Stickel 1991, Matsumoto 2002, Skinner and Sloan 2002). For modeling simplicity, we standardize loss aversion with respect to guidance to be 0: γ is the difference between guidance and earnings announcement loss aversion. This standardization can be relaxed: for model results to hold, it only needs to be true that investors experience a greater degree of loss aversion at t_1 relative to t_0. In effect, the manager trades off the net difference in loss aversion against interim stock prices.

Following canonical disclosure models, we focus on truthful disclosure. Sections 1 and 2 discuss the literature on earnings management, which we view as an important and complementary consideration of managers in their endeavor to beat benchmarks. A seperate and large literature on managerial reputation and litigation risk documents the incentives for managers to avoid cheap talk or bid-shaving. Stocken (2000b) finds that repeated games between managers and investors induces truthful disclosure. Beyer and Dye (2012) models how managers strategically trade off reputation of truth telling against incentives to withhold private signals. Furthermore, litigation risk disciplines firms from engaging in untruthful disclosures.[3]

Our baseline model assumes no discounting because of the short-horizon of the strategic disclosure. Managers strategically disclose to influence quarterly earnings expectations. Incorporating discount rates increases the weight managers place on interim prices and de-

[3] Johnson et al. (2001) document that legal protections from litigation do not decrease the quality of firm forward-looking disclosures. In the cross-section, firms at greater risk of litigation provide more comprehensive disclosures.

creases the weight on the risk of future loss aversion. In the model, the manager weighs the prices in two periods equally. This assumption intends to emphasize that the manager cares about interim price, thus cares about how disclosure affects the investors' perception of firm value and likelihood of disappointment. This equal weighting can be relaxed in favor of a more general assumption where the manager may assign different weights on the stock prices at each period: for example, he maximizes the expected value of $\lambda \cdot P_0 + (1 - \lambda) \cdot P_1$. Here, λ captures the degree of myopic focus on the interim price. As long as $\lambda > 0$, the trade-off in our baseline model persists. As λ increases, the manager cares more about the interim price effect and less about disappointing investors in the next period. As a result, we are more likely to converge towards the standard equilibrium result where managers with news above a certain threshold disclose. With sufficiently myopic managers, the cost of lower interim prices would outweigh the benefit of avoiding disappointment. On the other hand, as λ decreases, the manager is less myopic and we expect to observe more disclosures from managers with mild-bad news and more withholding of mildly good news. In both cases, the manager strategically discloses as the model predicts.

7. Concluding Remarks

The primary contribution of this paper is to theoretically model how managers strategically guide market expectations. The model applies a behavioral feature of investor utility to a classic voluntary disclosure setting. The modification to investor utility is well-founded in the earnings announcement literature and behavioral economics literature. Investor utility includes a disappointment term associated with missing earnings expectations. This parsimonious modification to the classic model of Dye (1985) rationally explains why managers may voluntarily disclose bad news and withhold good news.

In making this contribution, we propose further research on the intersection of behavioral economics and voluntary disclosure. Much may be learned by incorporating well-documented behavioral features of investors into classic disclosure settings. Doing so need not abandon rationality or Bayesian updating of signals. Investor inattention, loss-aversion, and information aversion are all features of behavioral economics and finance. Disclosure models may be further enriched by incorporating these features.

References

Aboody, D. and R. Kasznik (2000). CEO stock option awards and the timing of corporate voluntary disclosures. *Journal of Accounting and Economics 29*(1), 73–100.

Baginski, S. P., J. M. Hassell, and M. D. Kimbrough (2002). The effect of legal environment on voluntary disclosure: Evidence from management earnings forecasts issued in US and Canadian markets. *Accounting Review 77*(1), 25–50.

Barberis, N., M. Huang, and T. Santos (2001). Prospect Theory and Asset Prices. *Quarterly Journal of Economics 116*(1), 1–53.

Barberis, N. and R. Thaler (2003). A survey of behavioral finance. *Handbook of the Economic of Finance*, 1052–1121.

Barton, J. and P. J. Simko (2002). The balance sheet as an earnings management constraint. *The accounting review 77*(s-1), 1–27.

Bartov, E., D. Givoly, and C. Hayn (2002). The rewards to meeting or beating earnings expectations. *Journal of Accounting and Economics 33*(2), 173–204.

Benartzi, S. and R. H. Thaler (1995). Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics 110*(1), 73–93.

Bertomeu, J., I. Marinovic, S. J. Terry, and F. Varas (2022). The dynamics of concealment. *Journal of Financial Economics 143*(1), 227–246.

Beyer, A., D. A. Cohen, T. Z. Lys, and B. R. Walther (2010). The financial reporting environment: Review of the recent literature. *Journal of Accounting & Economics 50*(2/3), 296–343.

Beyer, A. and R. A. Dye (2012). Reputation management and the disclosure of earnings forecasts. *Review of Accounting Studies 17*(4), 877–912.

Beyer, A. and I. Guttman (2012). Voluntary Disclosure, Manipulation, and Real Effects. *Journal of Accounting Research 50*(5), 1141–1177.

Bhojraj, S., P. Hribar, M. Picconi, and J. McInnis (2009). Making sense of cents: An examination of firms that marginally miss or beat analyst forecasts. *Journal of Finance 64*(5), 2361–2388.

Brown, L. D. (2001). How Important Is Past Analyst Forecast Accuracy? *Financial Analysts Journal 57*(6), 44–49.

Burgstahler, D. and I. Dichev (1997). Earnings management to avoid earnings decreases and losses. *Journal of accounting and economics 24*(1), 99–126.

Cohen, L., C. Malloy, and Q. Nguyen (2020). Lazy prices. *The Journal of Finance 75*(3), 1371–1415.

Crawford, V. P. and J. Sobel (1982). Strategic Information Transmission. *Econometrica 50*(6), 1431–1451.

Darrough, M. N. and N. M. Stoughton (1990). Financial disclosure policy in an entry game. *Journal of Accounting and Economics 12*(1-3), 219–243.

Dittmann, I., E. Maug, and O. Spalt (2010). Sticks or Carrots?Optimal CEO Compensation When Managers Are Risk Averse. *Journal of Finance 65*(6), 2015–2050.

Dutta, S. and B. Trueman (2002). The Interpretation of Information and Corporate Disclosure Strategies. *Review of Accounting Studies 7*(1), 75–96.

Dye, R. A. (1985). Disclosure of Nonproprietary Information. *Journal of Accounting Research 23*(1), 123.

Dye, R. A. (1986). Optimal Monitoring Policies in Agencies. *The RAND Journal of Economics 17*(3), 339–350.

Einhorn, E. and A. Ziv (2012). Biased voluntary disclosure. *Review of Accounting Studies 17*(2), 420–442.

Feltham, G. A. and J. Z. Xie (1992). Voluntary financial disclosure in an entry game with continua of types. *Contemporary Accounting Research 9*(1), 46–80.

Gigler, F. (1994). Self-Enforcing Voluntary Disclosures. *Journal of Accounting Research 32*(2), 224.

Gigler, F. B. and T. Hemmer (2001). Conservatism, optimal disclosure policy, and the timeliness of financial reports. *The Accounting Review 76*(4), 471–493.

Graham, J. R., C. R. Harvey, and S. Rajgopal (2005). The economic implications of corporate financial reporting. *Journal of Accounting and Economics 40*(1), 3–73.

Grossman, S. J. (1981). An introduction to the theory of rational expectations under asymmetric information. *The Review of Economic Studies*.

Grossman, S. J. and O. D. Hart (1980). Takeover Bids, The Free-Rider Problem, and the Theory of the Corporation. *The Bell Journal of Economics 11*(1), 42.

Guttman, I., O. Kadan, and E. Kandel (2006). A rational expectations theory of kinks in financial reporting. *The Accounting Review 81*(4), 811–848.

Guttman, I., I. Kremer, and A. Skrzypacz (2014). Not only what but also when: A theory of dynamic voluntary disclosure. *American Economic Review 104*(8), 2400–2420.

Heidhues, P. and B. Kőszegi (2014). Regular prices and sales. *Theoretical Economics 9*(1), 217–251.

Johnson, M. F., R. Kasznik, and K. K. Nelson (2001). The impact of securities litigation reform on the disclosure of forward-looking information by high technology firms. *Journal of Accounting Research 39*(2), 297–327.

Johnson, T. L. and E. C. So (2018). Time will tell: Information in the timing of scheduled earnings news. *Journal of Financial and Quantitative Analysis 53*(6), 2431–2464.

Jovanovic, B. (1982). Selection and the Evolution of Industry Author(s): Boyan Jovanovic Source:. *Econometrica 50*(3), 649–670.

Jung, W.-O. and Y. K. Kwon (1988). Disclosure When the Market Is Unsure of Information Endowment of Managers. *Journal of Accounting Research 26*(1), 146.

Kahneman, D. and A. Tversky (1979). Kahneman & Tversky (1979) - Prospect Theory - An Analysis Of Decision Under Risk.pdf.

Kasznik, R. and B. Lev (1995). To Warn or Not to Warn: Management Disclosures in the Face of an Earnings Surprise To Warn or Not t o Warn: Management Disclosures i n the Face of an Earnings Surprise. *Source: The Accounting Review THE ACCOUNTING REVIEW 70*(1), 113–134.

Korn, E. (2004). Voluntary Disclosure of Partially Verifiable Information. *Schmalenbach Business Review 56*(April), 139–163.

Koszegi, B. and M. Rabin (2006). A Model of Reference-Dependent Preferences. *The Quarterly Journal of Economics 121*(4), 1133–1165.

Lanen, W. N. and R. E. Verrecchia (1987). Operating Decisions and the Disclosure of Management Accounting Information. *Journal of Accounting Research 25*(1987), 165–189.

Matsumoto, D. A. (2002). Management's incentives to avoid negative earnings surprises.

McNichols, M. F. and S. R. Stubben (2008). Does earnings management affect firms' investment decisions? *The accounting review 83*(6), 1571–1603.

Milgrom, P. R. (1981). Good News and Bad News: Representation Theorems and Applications. *The Bell Journal of Economics 12*(2), 380.

Pae, S. (2002). Optimal Disclosure Policy in Oligopoly Markets. *Journal of Accounting Research 40*(3), 901–932.

Penno, M. C. (1997). Information quality and voluntary disclosure. *Accounting Review*.

Richardson, S., S. H. Teoh, and P. D. Wysocki (2004). The walk-down to beatable analyst forecasts: The role of equity issuance and insider trading incentives. *Contemporary accounting research 21*(4), 885–924.

Ross, S. A. (1977). The Determination of Financial Structure: The Incentive-Signalling Approach. *The Bell Journal of Economics 8*(1), 23.

Shefrin, H. and M. Statman (1985). The disposition to sell winners too early and ride losers too long: Theory and evidence. *The Journal of finance 40*(3), 777–790.

Skinner, D. (1994). Why Firms Voluntarily Disclose Bad News Author (s): Douglas J . Skinner Published by : Wiley on behalf of Accounting Research Center , Booth School of Business , University of Chicago Stable URL : http://www.jstor.org/stable/2491386 JSTOR is a not-for-p. *32*(1), 38–60.

Skinner, D. J. and R. G. Sloan (2002). Earnings surprises, growth expectations, and stock returns or don't let an earnings torpedo sink your portfolio. *Review of accounting studies 7*(2-3), 289–312.

Soffer, L. C., L.-S. Edu, S. Ramu Thiagarajan, and B. R. Walther (2000). Earnings Preannouncement Strategies. *Review of Accounting Studies 5*(2000), 5–26.

Starmer, C. (2000). Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk. *Journal of Economic Literature 38*(2), 332–382.

Stein, J. C. (1989). Efficient capital markets, inefficient firms: A model of myopic corporate behavior. *The quarterly journal of economics 104*(4), 655–669.

Stickel, S. E. (1991). Common stock returns surrounding earnings forecast revisions: More puzzling evidence. *Accounting Review*, 402–416.

Stocken, P. C. (2000a). Credibility of Voluntary Disclosure. *The RAND Journal of Economics 31*(2), 359.

Stocken, P. C. (2000b). Credibility of voluntary disclosure. *The RAND Journal of Economics*, 359–374.

Suijs, J. (2007). Voluntary disclosure of information when firms are uncertain of investor response. *Journal of Accounting and Economics 43*(2-3), 391–410.

Tversky, A. and D. Kahneman (1992). Advances in prospect theory: Cummulative representation of uncertainty.

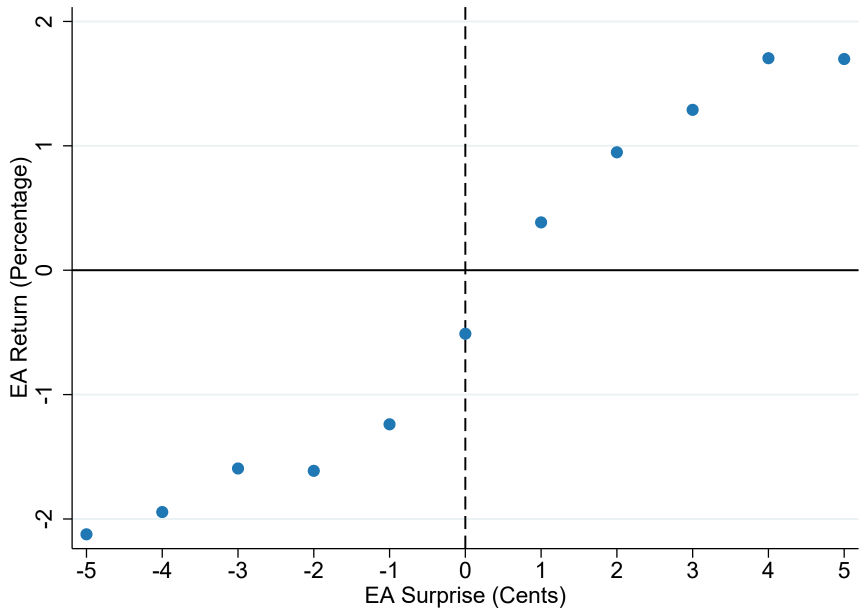
Verrecchia, R. (1983). Discretionary disclosure. *Journal of accounting and economics 5*, 179–194.

Verrecchia, R. E. (1990). Information quality and discretionary disclosure. *Journal of Accounting and Economics 12*(4), 365–380.

Figure 1. Asymmetric and Discontinuous Returns

Figure 1 illustrates the discontinuity in earnings announcement returns about analyst forecasts of earnings per share. Earnings announcement returns are 3-day returns centered about the announcement date. Panel A shows the average earnings announcement returns for the range of missing or beating by +5 / -5 cents. There is a 1.6 percent return difference between beating and missing by 1 cent. Panel B shows the average earnings announcement returns for a wider range of +0.5 / -0.5 percent (scaled by lagged share prices).

Panel A: Earnings Surprise in Cents



Panel B: Earnings Surprise in Percent

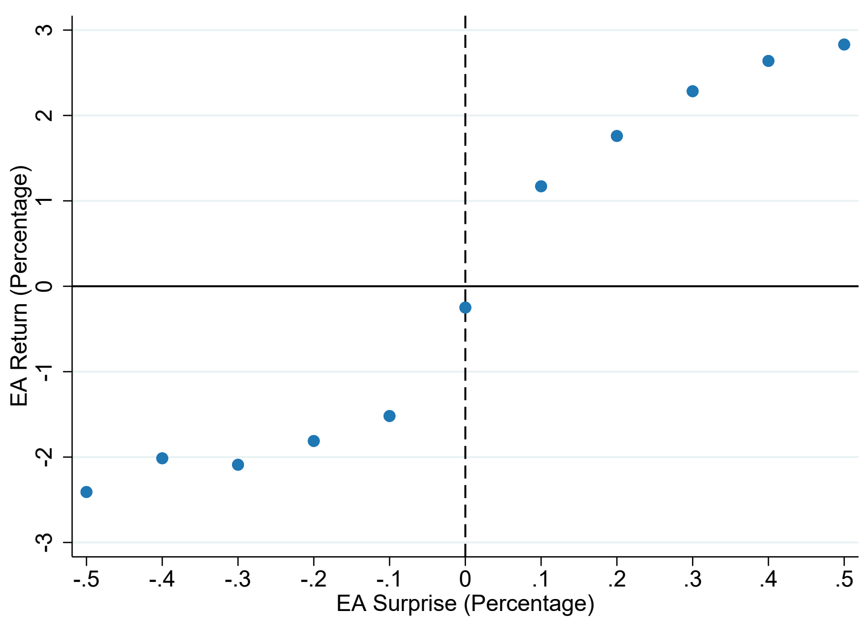


Figure 2. Distribution of Guidance Surprise and Earnings Surprise

Figure 2 illustrates the distribution of guidance and earnings surprise truncated at +/- 0.5%. *Guidance Surprise* is the difference between managerial guidance and prior consensus leading up to the guidance, scaled by the lagged share price (see Equation 1). Conditional on issuing earnings guidance, managers are much more likely to provide guidance that is lower than consensus. If we consider this narrow window around prior consensus, then managers are 73% likely to provide mildly negative guidance. Similarly, we construct *Earnings Surprise* as the difference between realized earnings and updated analyst consensus following managerial guidance, scaled by the lagged share price (Equation 2). The graph illustrates that, conditional on issuing earnings guidance, managers are much more likely to beat than to miss analyst expectations. If we consider this narrow window around consensus expectations, then managers are 72% likely to beat earnings expectations.

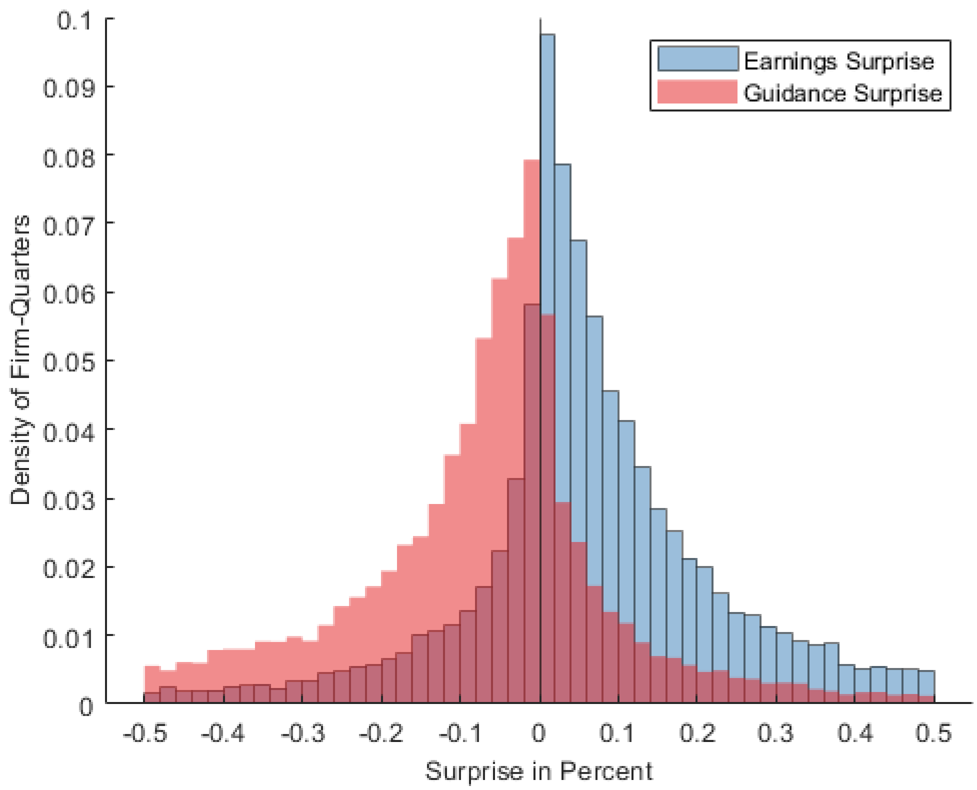


Figure 3. Time-line of Events

Figure 3 illustrates the time-line of events of the model. This is a static model that considers two points in time. In the beginning of the period, the manager receives, with probability q, a noisy signal s about future earnings x. The manager chooses whether to truthfully disclose or not disclose. The investor rationally Bayesian-updates his earnings expectation and prices the firm, P_0. At the end of the period, cash flows are realized and revealed to all. The investor observes the realized earnings and prices the firm, P_1.

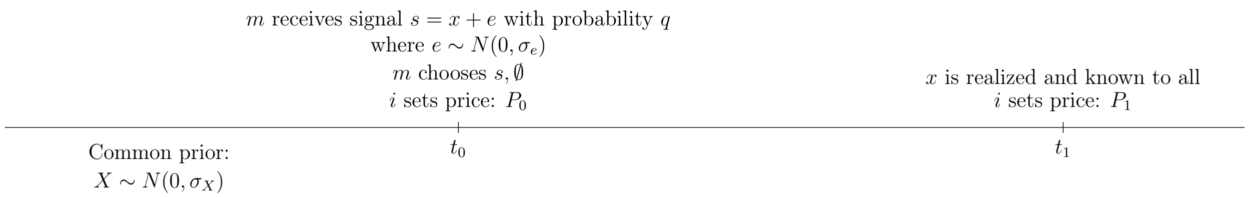
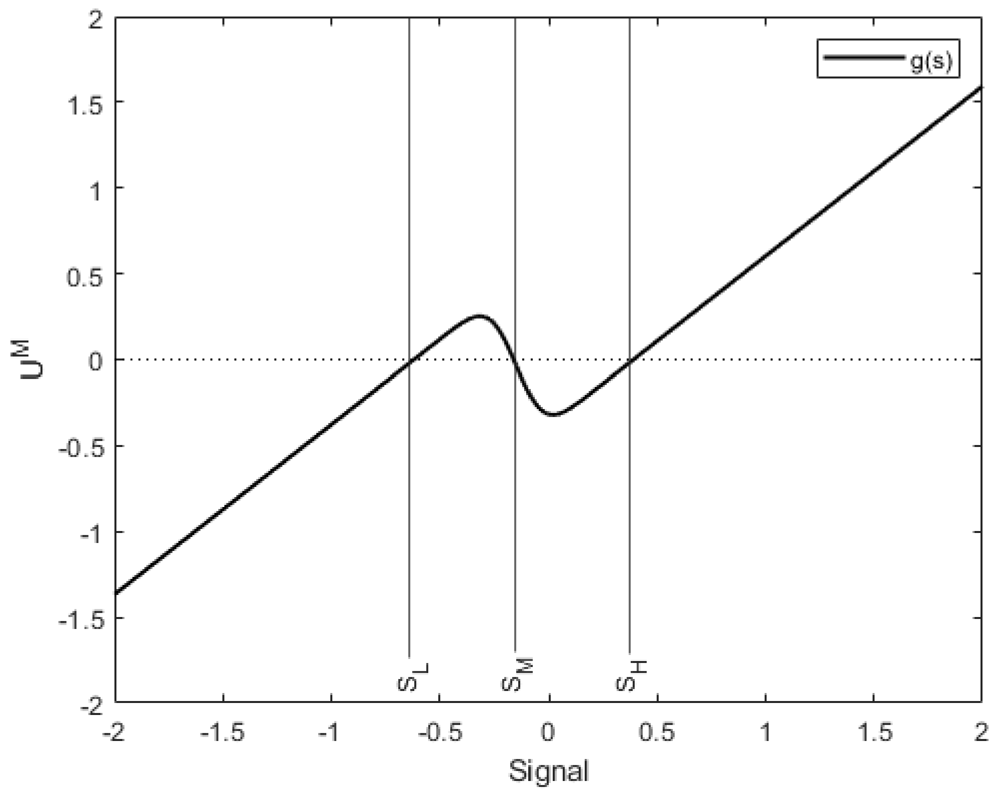


Figure 4. The Optimal Disclosure Rule

Figure 4 illustrates an example where the optimal disclosure rule is a 3-threshold (s_L, s_M, s_H) equilibrium. The manager who draws very bad news $(s < s_L)$ does not disclose. The interim price benefit from pooling with those who did not receive a signal exceeds expected future disappointment. The manager who draws mildly bad news $(s_L \leq s < s_M)$ discloses. The interim price decrease is worth the decrease in expected future disappointment. The manager who draws mildly positive news $(s_M \leq s < s_H)$ does not disclose. The interim price increase is not worth the greater expected future disappointment. Finally, the manager who draws very good news $(s_H \leq s)$ does disclose. The interim price benefit exceeds the increase in expected future disappointment.

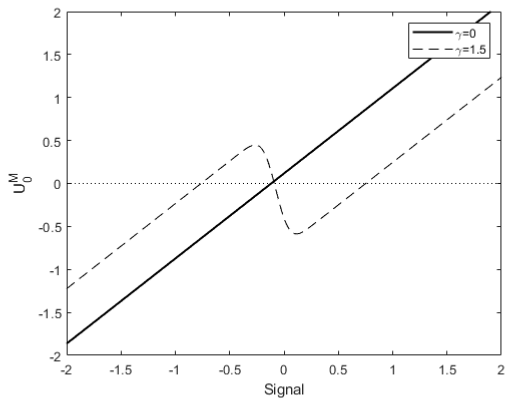


Parameters: $q = 0.5, \sigma_X = 1, \sigma_e = 0.1, \gamma = 1$

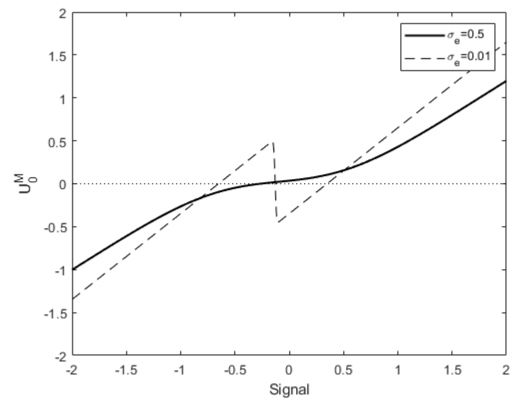
Figure 5. The Optimal Disclosure Rule

Figure 5 contains three panels that illustrate the effect of varying disappointment, signal noise, and probability of receiving a signal on the optimal disclosure rule. The benchmark case is shown in Figure 4. Panel A compares no investor loss aversion with high investor loss aversion. Panel B displays the reporting rule with noisy vs. precise signals. Panel C displays the reporting rule with high vs. low probability of having received a signal.

Panel A: Dye (1985) vs. Large Loss Aversion



Panel B: Noisy vs. Precise Signal



Panel C: High vs. Low Probability of Signal

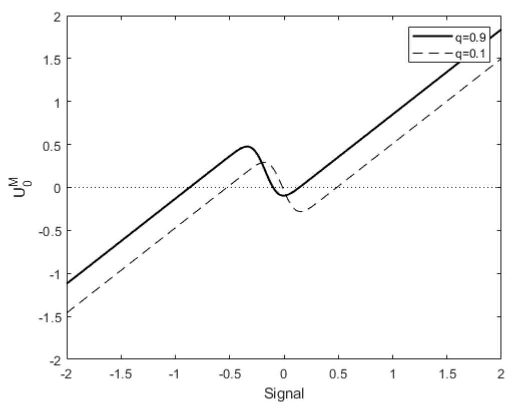
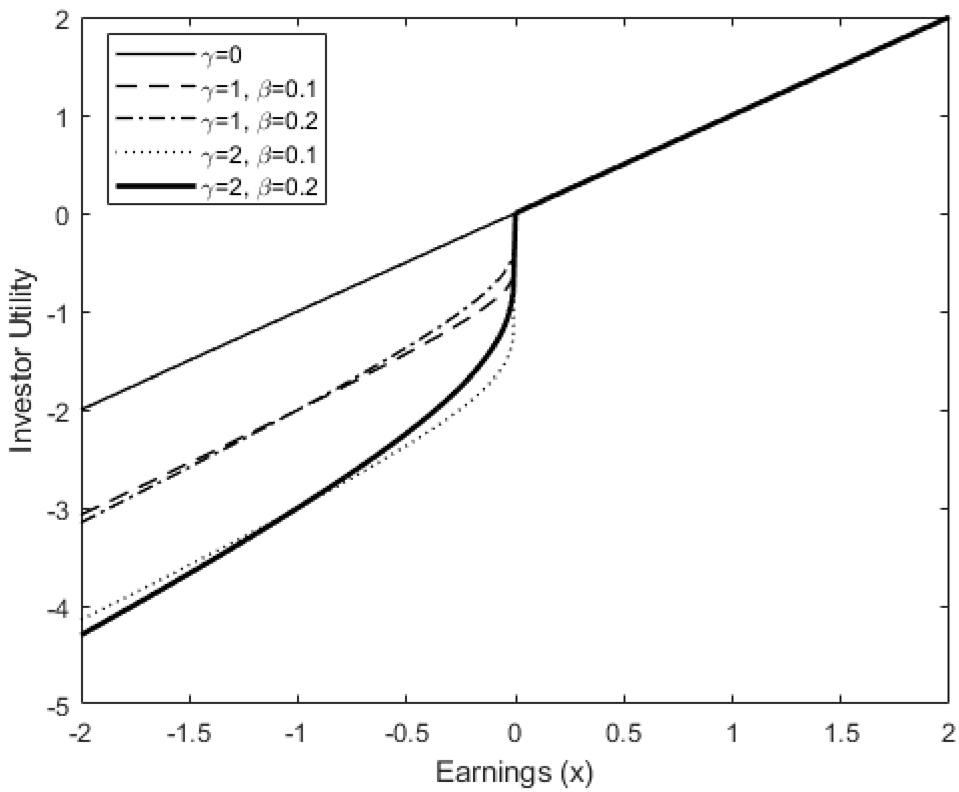


Figure 6. **Cumulative Prospect Theory Utility Function**

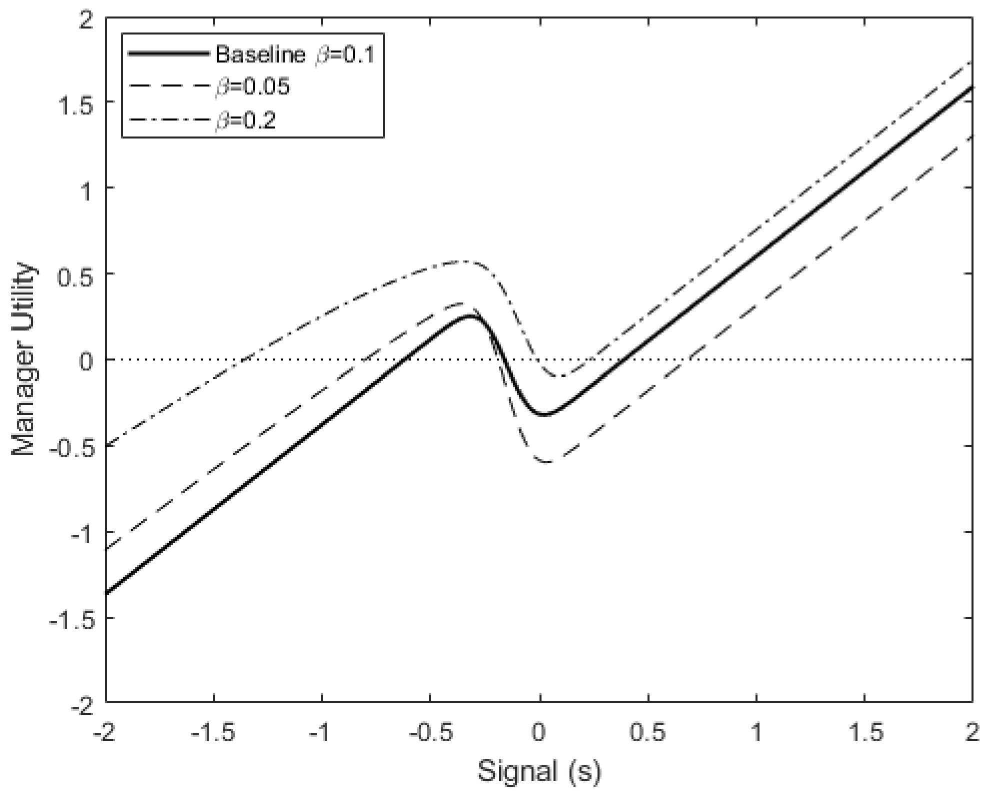
Figure 6 features a more general Prospect-Theoretic utility function. Investor utility is continuous and exhibits diminishing marginal sensitivity to loss-aversion. As γ increases, the investor is more loss-averse: earnings below expectations are farther from the linear-utility benchmark. As β increases, the curvature of investor utility is flatter. Marginal loss-aversion asymptotes to 0 more slowly for larger β.



Parameters: $q = 0.5, \sigma_X = 1, \sigma_e = 0.1, \gamma = 1$

Figure 7. **Optimal Disclosure Rule under Prospect-Theoretic Investor Utility**

Figure 7 illustrates the optimal disclosure rule of managers facing investors with more general Prospect-Theoretic utility. The economic forces remain the same, resulting in a three-threshold optimal disclosure strategy. Steeper investor loss-aversion decreases the range of strategic disclosure. Disclosure of mildly negative news decreases and withholding of mildly positive news increases. The economic intuition is that marginal loss aversion asymptotes to 0 quickly for steep loss-aversion (low β), which decreases the incentive to disclose negative news. In contrast, small misses to earnings bear large loss aversion effects, which increases the incentive to withhold mildly good news. The opposite effects are true for flatter investor loss-aversion.



Parameters: $q = 0.5, \sigma_X = 1, \sigma_e = 0.1, \gamma = 1$

1. Appendix

1.1. Proof of Proposition 1

We use the Brouwer fixed-point theorem. Define the function $h(\kappa, \delta) \rightarrow (\kappa', \delta')$ as a mapping, where κ is the investor's conjecture about the non-disclosure firms' average value and δ the expected probability of disappointment. Given (κ, δ), managers form their best response disclosure strategy, resulting a set of non-disclosure firms with an average value of κ' and disappointment probability of δ'. We first show that it is without loss of generality to assume that the domain of (κ, δ) is compact, i.e., closed and bounded. It is easy to see $\delta \in [0, 1]$. Regarding κ, we consider the minimum expected firm value conditional on a disclosure strategy. We can show that his value is bounded below using the minimum principle (e.g., Guttman et al. (2014)). Since the domain is compact and convex, there must exist a fixed point (κ^*, δ^*) such that $h(\kappa^*, \delta^*) = (\kappa^*, \delta^*)$.

Suppose the expected $\hat{E}(x|\emptyset) = \kappa$ and $\hat{P}(x < \kappa) = \delta$. Given this belief, the firm chooses to disclose if and only if:

$$g(s, \kappa, \delta) \equiv P(\hat{r} = s) - P(\hat{r} = \emptyset) \geq 0$$

where

$$g(s, \kappa, \delta) = E(x|s) - (E(x|\emptyset) + \gamma(\delta + \Pr(x < \kappa|s)) - 1) \tag{12}$$

Note that this function is continuous and differentiable with respect to κ and δ. To ease notation, we denote $\eta = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2}$, denote $\sigma_{x,s} = \sqrt{\frac{1}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_s^2}}}$, and Φ to be the c.d.f. of a standard normal distribution. We have:

$$
\begin{aligned}
E\left(x|s\right) &= \frac{\sigma_x^2}{\sigma_x^2+\sigma_\epsilon^2}s = \eta s \\
\Pr\left(x<\kappa|s\right) &= \int_{-\infty}^{\kappa} f\left(x|s\right)dx \\
&= \Phi\left(\frac{\kappa-\eta s}{\sigma_{x,s}}\right)
\end{aligned}
$$

Take the s^* as the roots of Equation 12. Firms with s^* would be indifferent between disclosing or withholding, and without loss of generality, we assume that they disclose. Since their mass is 0, this assumption does not affect the investor's inference.

$$
g\left(s^*,\kappa,\delta\right) = 0
$$

There would be at least 1 and at most 3 roots. To see this, note

$$
\begin{aligned}
g\left(s,\kappa,\delta\right) &= \eta s-\kappa+\gamma\left(\delta+\Phi\left(\frac{\kappa-\eta s}{\sigma_{x,s}}\right)-1\right) \\
&= \eta s-\kappa+\gamma\left(\delta-\Phi\left(\frac{\eta s-\kappa}{\sigma_{x,s}}\right)\right)
\end{aligned}
$$

Thus

$$
\begin{aligned}
g'\left(s\right) &= \eta\left(1-\frac{\gamma}{\sigma_{x,s}}\phi\left(\frac{\eta s-\kappa}{\sigma_{x,s}}\right)\right) \\
g''\left(s\right) &= \left(\frac{\eta s-\kappa}{\sigma_{x,s}}\right)\frac{\eta}{\sigma_{x,s}}\frac{\gamma}{\sigma_{x,s}}\phi\left(\frac{\eta s-\kappa}{\sigma_{x,s}}\right)
\end{aligned}
$$

Thus $s^*\left(\kappa,\delta\right)$ is continuous in κ and δ, since $\frac{\partial s^*}{\partial\kappa} = -\frac{\partial g}{\partial\kappa}/\frac{\partial g}{\partial s}|_{(s^*,\kappa,\delta)}$ and $\frac{\partial s^*}{\partial\delta} = -\frac{\partial g}{\partial\delta}/\frac{\partial g}{\partial s}|_{(s^*,\kappa,\delta)}$.

If there is only one root,

$$\Pr\left(x<\kappa|g\left(s,\kappa,\delta\right)<0\right)=\int_{-\infty}^{s^{*}}f\left(s\right)ds$$
$$E\left(x|g\left(s,\kappa,\delta\right)<0\right)=\frac{\int_{-\infty}^{s^{*}}\eta sf\left(s\right)ds}{\int_{-\infty}^{s^{*}}f\left(s\right)ds}$$

If there are three roots, then

$$\Pr\left(x<\kappa|g\left(s,\kappa,\delta\right)<0\right)=\int_{-\infty}^{s_{L}^{*}}f\left(s\right)ds+\int_{s_{M}^{*}}^{s_{H}^{*}}f\left(s\right)ds$$
$$E\left(x|g\left(s,\kappa,\delta\right)<0\right)=\frac{\int_{-\infty}^{s_{L}^{*}}\eta sf\left(s\right)ds+\int_{s_{M}^{*}}^{s_{H}^{*}}\eta sf\left(s\right)ds}{\int_{-\infty}^{s_{L}^{*}}f\left(s\right)ds+\int_{s_{M}^{*}}^{s_{H}^{*}}f\left(s\right)ds}$$

Either case, $E\left(x<\kappa|\emptyset,\kappa,\delta\right)$ and $\Pr\left(x<\kappa|\emptyset,\kappa,\delta\right)$ are continuous in s^{*} or s_{L}^{*}, s_{M}^{*}, s_{H}^{*}, which in turn are continuous in κ and δ.

Next, we show that $E\left(x<\kappa|I=\emptyset,\kappa,\delta\right)$ is bounded below. If $g\left(s,\kappa,\delta\right)=0$ has only one root, then firms with $s<s^{*}$ withholds. Let x follow a normal distribution with mean :

$$E\left(x<\kappa|\emptyset,\kappa,\delta\right)=\frac{q\int_{-\infty}^{s^{*}}\eta sf\left(s\right)ds}{1-q+q\int_{-\infty}^{s^{*}}f\left(s\right)ds}\geq\liminf_{y}\frac{q\int_{-\infty}^{y}\eta sf\left(s\right)ds}{1-q+q\int_{-\infty}^{y}f\left(s\right)ds}$$

Since $\frac{q\int_{-\infty}^{y}\eta sf(s)ds}{1-q+q\int_{-\infty}^{y}f(s)ds}$ first decreases and then increases with y, it is bounded below.

If $g\left(s,\kappa,\delta\right)=0$ has three roots, then firms with $s<s_{L}^{*}$ and $s_{M}^{*}<s<s_{H}^{*}$withholds, then

we can always find s' such that $s' < s_H^*$ and $\int_{s_L^*}^{s'} f(s)\,ds = \int_{s_M^*}^{s_H^*} f(s)\,ds$. Then

$$\begin{aligned} E\left(x|g\left(s,\kappa,\delta\right)<0\right) &= \frac{\int_{-\infty}^{s_L^*} \eta s f(s)\,ds + \int_{s_M^*}^{s_H^*} \eta s f(s)\,ds}{\int_{-\infty}^{s_L^*} f(s)\,ds + \int_{s_M^*}^{s_H^*} f(s)\,ds} \\ &> \frac{\int_{-\infty}^{s_L^*} \eta s f(s)\,ds + \int_{s_L^*}^{s'} \eta s f(s)\,ds}{\int_{-\infty}^{s_L^*} f(s)\,ds + \int_{s_L^*}^{s'} f(s)\,ds} \\ &\geq \liminf_{y} \frac{q\int_{-\infty}^{y} \eta s f(s)\,ds}{1-q+q\int_{-\infty}^{y} f(s)\,ds} \end{aligned}$$

Thus we can see that $E\left(x|\kappa,\delta\right)$ is bounded below. It is also bounded above by 0, the unconditional sample mean. Also note that $\delta \in [0,1]$. Furthermore, the mapping from (κ,δ) to $\left(E\left(x<\kappa|\emptyset,\kappa,\delta\right), \Pr\left(x<\kappa|\emptyset,\kappa,\delta\right)\right)$ is continuous, we can use the fixed-point theorem and thus there exist at least one equilibrium (κ^*,δ^*) such that $\Pr\left(x<\kappa^*|\emptyset,\kappa^*,\delta^*\right)=\delta^*$ and $E\left(x<\kappa^*|\emptyset,\kappa^*,\delta^*\right)=\kappa^*$.

Q.E.D.

1.2. Proof of Lemma 1

We prove that the maximum number of thresholds is 3 by showing that $g(s)$ is cubic in s and applying the fundamental theorem of algebra. We know that $g(s)$ is cubic because $g'(s)$ is quadratic in s, where $\eta = \frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2}$:

$$
\begin{aligned}
g'(s) &= \eta\left(1 - \frac{\gamma}{\sigma_{x,s}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{E[x|\emptyset]-\eta s}{\sigma_{x,s}}\right)^2}\right) \\
g''(s) &= -\eta \frac{\gamma}{\sigma_{x,s}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{E[x|I=\emptyset]-\eta s}{\sigma_{x,s}}\right)^2}\left(-\frac{E[x|\emptyset]-\eta s}{\sigma_{x,s}}\right)\left(\frac{1}{\sigma_{x,s}}\cdot\eta\right) \\
&= \frac{1}{2}\eta^2\left(\frac{E[x|\emptyset]-\eta s}{\sigma_{x,s}}\right)\frac{\gamma}{\sigma_{x,s}^2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{E[x|\emptyset]-\eta s}{\sigma_{x,s}}\right)^2} \\
g''\left(\beta E[x|\emptyset]\right) &= 0 \\
\implies &\begin{cases} g''(s) > 0 & \text{for } s < \frac{1}{\eta}E[x|\emptyset] \\ g''(s) < 0 & \text{for } s > \frac{1}{\eta}E[x|\emptyset] \end{cases}
\end{aligned}
$$

Therefore, $g(s)$ is cubic in s.

Q.E.D.

1.3. Proof of Proposition 2

In this proof, we characterize the three-threshold equilibrium. We first conjecture the equilibrium and then verify it satisfies the equilibrium condition. Suppose that the manager receives a signal s, and the investor's conjecture about non-disclosure firms is that their average firm value is κ and probability of disappointment is δ. The manager Bayesian-updates the distribution of expected cash flows:

$$E[\tilde{x}|s] = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2} s$$
$$\sigma_{x,s}^2 = \frac{\sigma_e^2 \sigma_X^2}{\sigma_e^2 + \sigma_X^2}$$
$$f(x|s) = N(E[\tilde{x}|s], \sigma_{x,s}^2)$$

where $\phi(x)$ is the standard normal PDF (and $\Phi(x)$ denotes the standard normal CDF).

If the manager chooses to <u>disclose the signal (s)</u>:

$$P_0(s) = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2} s - \gamma \int_{-\infty}^{E[\tilde{x}|s]} f(x|s)dx = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2} s - \frac{\gamma}{2}$$

Note that prices are martingale for both the manager and investors when there is no asymmetric information:

$$E[P_1(s)] = P_0(s) = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2} s - \frac{\gamma}{2}$$

If the manager receives a signal but chooses not to disclose ($I = \emptyset$):

$$P_0(\emptyset) = E[\tilde{x}|\emptyset] - \gamma Pr[\text{Disappointment}|\emptyset]$$

where

$$E[x|I=\emptyset] = A \cdot 0 + B \cdot \frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2} E[s|\emptyset]$$

$$A = \frac{1-q}{1-q+qPr(\emptyset|s)}$$

$$B = \frac{qPr(\emptyset|s)}{1-q+qPr(\emptyset|s)}$$

Note $\sigma_s^2 = \sigma_X^2 + \sigma_e^2$

$$\begin{aligned} Pr(\emptyset|s) &= \Phi\left(\frac{s_L}{\sigma_s}\right) + \Phi\left(\frac{s_H}{\sigma_s}\right) - \Phi\left(\frac{s_M}{\sigma_s}\right) \\ &= \int_{-\infty}^{s_L} \frac{1}{\sigma_s\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{s}{\sigma_s})^2} ds + \int_{s_M}^{s_H} \frac{1}{\sigma_s\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{s}{\sigma_s})^2} ds \end{aligned}$$

and

$$\begin{aligned} E[\text{Disappointment}|\emptyset] =& -A \cdot \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x)dx - B \cdot \gamma \left(\int_{-\infty}^{s_L} f(s)ds \cdot \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s \leq s_L)dx \right. \\ & \left. + \int_{s_M}^{s_H} f(s)ds \cdot \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s_M \leq s < s_H)dx \right) \\ =& -A \cdot \gamma \cdot \Phi\left(\frac{E[\tilde{x}|\emptyset]}{\sigma_X}\right) - B \cdot \gamma \left[\Phi\left(\frac{s_L}{\sigma_s}\right) \cdot \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s \leq s_L)dx \right. \\ & \left. + \left(\Phi\left(\frac{s_H}{\sigma_s}\right) - \Phi\left(\frac{s_M}{\sigma_s}\right) \right) \cdot \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s_M \leq s < s_H)dx \right] \end{aligned}$$

The expected earnings conditional on the manager drawing a *very bad* signal:

$$E\left[X|s \leq s_L\right] = \frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2} E[s|s \leq s_L] = -\frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2} \left(\frac{\sigma_s}{\sqrt{2\pi}} e^{\frac{1}{2}(\frac{s_L}{\sigma_s})^2} \right) / \left(Pr(s \leq s_L)\right)$$

The expected earnings conditional on the manager drawing a *mildly good* signal:

$$E[X|s_M \leq s < s_H] = \frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2} E[s|s_M \leq s < s_H] = \frac{\left(\frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2}\right)\left(\frac{\sigma_s}{\sqrt{2\pi}}\right)\left(e^{\frac{1}{2}(\frac{s_M}{\sigma_s})^2} - e^{\frac{1}{2}(\frac{s_H}{\sigma_s})^2}\right)}{Pr(s_M \leq s \leq s_H)}$$

Then, knowing the variance of the expected earnings conditional on the manager receiving a private signal: $\sigma^2_{x,s} = \frac{\sigma^2_e \sigma^2_X}{\sigma^2_e + \sigma^2_X}$.

At $t = 1$,

$$E[P_1|\emptyset] = \frac{\sigma^2_X}{\sigma^2_X + \sigma^2_e} s - \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx$$
$$\int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx = \Phi\left(\frac{E[\tilde{x}|\emptyset] - \frac{\sigma^2_X}{\sigma^2_X+\sigma^2_e}s}{\sigma_{x,s}}\right)$$

Finally, if the manager does not receive a signal, he cannot make a disclosure and cannot credibly signal that he has not received the signal. (I=$\emptyset$).

Let the function, $g(s,\kappa,\delta)$, denote the difference in the manager's utility U^m between disclosing and not disclosing the private signal:

$$\begin{aligned} g(s,\kappa,\delta) &= U^m(s) - U^m(\emptyset) \\ &= (P_0(s) - P_0(\emptyset)) - \left(\frac{\gamma}{2} - \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx\right) \end{aligned}$$

$\because \qquad \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx \leq 1$

$\therefore \qquad lim_{s\to-\infty} g(s) = (-\infty - P_0(\emptyset)) - \frac{\gamma}{2} + \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx \leq -\infty - P_0(\emptyset) + \frac{\gamma}{2} = -\infty$

Note that

$$lim_{s\to-\infty} \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx = \gamma$$

The manager optimally does not choose disclosure for any $s \to -\infty$

$\because \qquad \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx \geq 0$

$\therefore \qquad lim_{s\to\infty} g(s) = (\infty - P_0(\emptyset) - \frac{\gamma}{2} + \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx \geq (\infty - P_0(\emptyset)) - \frac{\gamma}{2} = \infty$

Note that

$$lim_{s\to\infty}\gamma\int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx = 0$$

The manager optimally chooses to disclose for any $s \to \infty$

Since $g(s)$ is continuous, then by the Intermediate Value Theorem: $\exists s'$ such that $g(s') = 0$ Thus, we have proven that $\exists$ at least one threshold, s', without any parameter restrictions.

For given $\kappa = E(x|\emptyset)$ and $\delta = Pr(Disappointment|\emptyset)$, if $\exists s^\star$ s.t. $g'(s^\star) < 0$ and $g(s^\star) = 0$, and three roots exist, with $s_L \in (-\infty, s^\star)$, $s_M = s^\star$, and $s_H \in (s^\star, \infty)$ because of the Intermediate Value Theorem: $g(-\infty) = -\infty$ and $g(s^\star - \epsilon) > 0$, $g(s^\star) = 0$, and $g(s^\star + \epsilon) < 0$ and $g(\infty) = \infty$, for any arbitrarily small e. Thus, it is a three-threshold equilibrium if the non-disclosure firms, either uninformed or with $s \in -\infty, s_L$ or $s \in (s_M, s_H)$, satisfies the condition that their expected value is κ and disappointment probability is δ.

Thus, when a three-threshold equilibrium exists, the threshold points satisfy:

$$g(s) = (P_0(s) - P_0(\emptyset)) - \left(\frac{\gamma}{2} - \gamma\int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx\right) = 0$$

and the middle threshold point has:

$$g'(s) = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2}\left(1 - \frac{\gamma}{\sigma_{x,s}}\phi\left(\frac{E[\tilde{x}|\emptyset] - \frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2}s}{\sigma_{x,s}}\right)\right) < 0$$

Q.E.D.

1.4. Proof of Proposition 3

Note that first-order derivative $g'(s)$ gives:

$$g(s) = (P_0(s) - P_0(\emptyset)) - \left(\frac{\gamma}{2} - \gamma \int_{-\infty}^{E[\tilde{x}|\emptyset]} f(x|s)dx\right)$$
$$g'(s) = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2}\left(1 - \frac{\gamma}{\sigma_{x,s}} f\left(\frac{E[\tilde{x}|\emptyset] - \frac{\sigma_X^2}{\sigma_X^2+\sigma_e^2}s}{\sigma_{x,s}}\right)\right)$$

Note that $f(\cdot) \leq \frac{1}{\sqrt{2\pi}}$, thus if $1 - \frac{\gamma}{\sigma_{x,s}\sqrt{2\pi}} > 0$, then $g'(s) > 0$ for any s. In that case, $g(s)$ monotonically increases and can have only one root with $g(s) = 0$. In that case, the three-threshold equilibrium does not exist. Thus, a necessary but insufficient condition for three roots to exist is that $1 - \frac{\gamma}{\sigma_{x,s}\sqrt{2\pi}} < 0$.

Also, we know by the Intermediate Value Theorem that at least one threshold exists. Since

$$\begin{array}{c} lim_{s\to-\infty} g(s) = -\infty \\ lim_{s\to+\infty} g(s) = +\infty \end{array}.$$

If $1 - \frac{\gamma}{\sigma_{x,s}\sqrt{2\pi}} > 0$ then $g'(s) > 0$ $\forall s$. Since $g(s)$ monotonically increases in s, this threshold is unique. In that case, we solve a one-threshold equilibrium by studying the indifference condition for the firm on the unique threshold.

Q.E.D.

1.5. Proof of Proposition 4

We prove this proposition by showing that the single-threshold equilibrium does not exist under these conditions. Given that Proposition 1 has established the existence of at least one equilibrium, it, therefore, follows that a three-threshold equilibrium exists in this case. Suppose a single-threshold equilibrium exists, in which the investor's expectation about firm value is κ, and the probability of disappointment is δ. By continuity, since $g(s, \kappa, \delta)$ is negative infinity when s goes to $-\infty$ and positive infinity when s goes to $+\infty$, it must be that the firm withholds below the threshold s^* and discloses otherwise, and $g'(s^*) > 0$.

$$\begin{aligned} g(s,\kappa,\delta) &= P(I=s) - P^i(I=\emptyset) \\ &= E(\tilde{x}|s) - E(\tilde{x}|\emptyset) + \gamma\left(\Pr(x<\kappa|s) + \delta - 1\right) > 0 \end{aligned}$$

Thus, the slope at the threshold has:

$$\begin{aligned} \frac{\partial g}{\partial s} &= \eta - \gamma \frac{\partial\left(\Pr(x<\kappa|s)+\delta-1\right)}{\partial s} \\ &= \eta - \gamma \frac{\partial \Phi\left(\frac{\kappa - E(\tilde{x}|s)}{\sigma_{x,s}}\right)}{\partial s} \\ &= \eta\left(1 - \frac{\gamma}{\sigma_{x,s}}\phi\left(\frac{\kappa - E(\tilde{x}|s)}{\sigma_{x,s}}\right)\right) \end{aligned}$$

Since $\phi(\cdot) \leq \left(0, \frac{1}{\sqrt{2\pi}}\right)$, if $\frac{\gamma}{\sigma_{x,s}}$ is sufficiently large and $\kappa = E(\tilde{x}|\emptyset)$ is sufficiently close to $E(\tilde{x}|s)$ then we have $\frac{\partial g}{\partial s} < 0$. This is the case where q is sufficiently small: Note that as $q \to 0$, the investor's belief would have $\kappa \to 0$ and $s^* \to 0$. That is, if the manager is most likely uninformed, the investor's expectation about non-disclosure firms is that their expected value is 0, and the threshold firm would have $s^* = 0$. This implies that firms with signal s right above (below) the threshold will withhold (disclose), which contradicts the single-threshold equilibrium. Since we already show that an equilibrium κ^*, δ^* exists (Proposition 3) and that in equilibrium, there is either one-threshold or three-threshold equilibrium (Lemma 1), then it must be the case that under the identified condition, the equilibrium has three thresholds, as identified in Proposition 2 .

Q.E.D.